Latent Predicate Networks: Concept Learning with Probabilistic Context-Sensitive Grammars

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Abstract

For humans, learning abstract concepts and learning language go hand in hand: we acquire abstract knowledge primarily through linguistic experience, and acquiring abstract concepts is a crucial step in learning the meanings of linguistic expressions. Number knowledge is a case in point: we largely acquire concepts such as seventy-three through linguistic means, and we can only know what the sentence "seventy-three is more than twice as big as thirty-one" means if we can grasp the meanings of its component number words. How do we begin to solve this problem? One approach is to estimate the distribution from which sentences are drawn, and, in doing so, infer the latent concepts and relationships that best explain those sentences. We present early work on a learning framework called Latent Predicate Networks (LPNs) which learns concepts by inferring the parameters of probabilistic context-sensitive grammars over sentences. We show that for a small fragment of sentences expressing relationships between English number words, we can use hierarchical Bayesian inference to learn grammars that can answer simple queries about previously unseen relationships within this domain. These generalizations demonstrate LPNs' promise as a tool for learning and representing conceptual knowledge in language.

Introduction

Although concept learning and language acquisition have typically been treated as distinct problems in AI, linguistics and cognitive development, they are strongly coupled for a child learning to understand language. More generally, people learn many abstract concepts primarily through language even though understanding language depends on understanding the underlying concepts. Research in concept learning is often focused on concepts grounded in perceptual features, and while it is almost certainly true that many concepts are learned via generalization from concrete examples, some concepts cannot be learned this way.

Number concepts are a good example: children do not learn about the meaning of "seventy five" by seeing examples of seventy five things; they do not know that "seventy five" is more than "twenty five" because of their perceptual experiences of these quantities. Rather, children learn the meaning of "seventy five" (or "a billion and five") by noticing how number words are used in language, in counting sequences, in arithmetic exercises, *etc*. Other good examples of such abstract concepts are kinship and social relations (*e.g.* "my father in law's grandmother"), temporal relations ("the day after last Thanksgiving."), and spatial relations ("above and just to the left of").

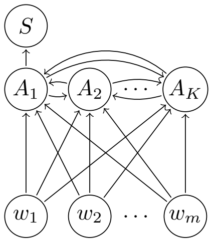
Such concepts share many of the properties of language syntax: they are unbounded in number, they derive their meanings via composition, and, although people only ever say, hear, or read about a small number of them, they are able to reason correctly about any them. There seems to be a grammar to these concepts, and grasping this grammar is critical to understanding their meanings and how to use them. This motivates our approach here, which is to apply the tools of probabilistic grammars more familiar from studies of syntax to the problem of concept aquisition.

Doing so requires overcoming some technical barriers.

First, whereas context-free grammars are suitable for describing large swathes of language syntax, the grammars of concepts are not context-free. To address this, we use Range Concatenation Grammars, a context-sensitive grammar formalism – one of several developed within linguistics – and extend this formalism to a probabilistic model of sentences.

Second, the categories of syntax – the nonterminals of the grammar – are often assumed to be known to children innately and given to automated learners by human experts. The categories that underlie conceptual knowledge, on the other hand, are far more numerous, vary from domain to domain, and are unlikely to be known to the learner. This motivates our use of latent predicates that, through learning, assume the role of a domain's underlying concepts (in the number domain, these might correspond to the concepts of successorship, order of magnitude, magnitude comparison, exact vs. approximate, *etc*.).

Finally, inducing probabilistic context-sensitive grammars with latent predicates threatens to be intractable: our goal is to find a middle ground between expressivity and tractability. Using PRISM (Sato and Kameya 2001) – a probabilistic logic programming system that naturally implements efficient dynamic programming algorithms for our models – we are able to explore which domains and which



$$\forall i,j,k \in [1,K];\ \forall a,b \in [1,m]$$
$$S(XY) \leftarrow A_1(X,Y).$$
$$A_i(XY,UV) \leftarrow A_j(X,Y), A_k(U,V).$$
$$A_i(XY,UV) \leftarrow A_j(X,U), A_k(Y,V).$$
$$A_i(XY,UV) \leftarrow A_j(X,U), A_k(V,Y).$$
$$\ldots$$
$$A_i(w_a,w_b).$$
$$m = |T|, K = |N \backslash S|, \mathcal{A}(A_i) = 2, \mathcal{A}(S) = 1$$

Figure 1: The architecture and rules of a schematic LPN.

grammar architectures are a good fit for this grammar-based approach.

The rest of this paper describes early work on this approach. First, we present Latent Predicate Networks (LPNs), our probabilistic model of concept learning. Then, we describe our approach to inference and its implementation in PRISM. Finally, we present preliminary experimental results in the domain of number concept learning that demonstrate the promise of this approach.

Latent Predicate Networks

An LPN is a hierarchical Bayesian model of strings extending the Hierarchical Dirichlet PCFG model (HD-PCFG) to Probabilistic Range Concatenation Grammars (PRCGs).

Probabilistic Range Concatenation Grammars

Range Concatenation Grammars (RCGs) are a class of string grammars that represent all and only those languages that can be parsed in time polynomial in the length of the target string (Boullier 2005). An RCG $G = (N, T, V, P, S)$ is a 5-tuple where N is a finite set of predicate symbols, T is a set of terminal symbols, V is a set of variable symbols, P is a finite set of $M \geq 0$ clauses of the form $\psi_0 \rightarrow \psi_1 \ldots \psi_M$, and $S \in N$ is the *axiom*. Each ψ_m is a term of the form $A(\alpha_1, \ldots, \alpha_{\mathcal{A}(A)})$, where $A \in N$, $\mathcal{A}(A)$ is the arity of A, and each $\alpha_i \in (T \cup V)^*$ is an argument of ψ_m. We call the left hand side term of any clause the *head* of that clause and its predicate symbol is the *head predicate*.

A string x is in the language defined by an RCG if one can *derive* $S(x)$. A derivation is a sequence of rewrite steps in which substrings of the left hand side argument string are bound to the variables of the head of some clause, thus determining the arguments in the clause body. If a clause has no body terms, then its head is derived; otherwise, its head is derived if its body clauses are derived.[1]

We extend RCGs to PRCGs by annotating each clause $C_k \in P$ with probabilities p_k such that for all predicates $A \in N$, $\sum_{k:head(C_k)=A} p_k = 1$. A PRCG defines a distribution over strings x by sampling from derivations of $S(x)$ according to the product of probabilities of clauses used in that derivation. This is a well defined distribution as long as no probability mass is placed on derivations of infinite length; in this paper, we only consider PRCGs with derivations of finite length, so we need not worry about this requirement. See Figure 2a for an example of the context-sensitive 2-copy language $\{ww \,|w \in \{a,b\}^+\}$ as a PRCG.

[1]This description of the language of an RCG technically only holds for *non-combinatory* RCGs, in which the arguments of body terms can only contain single variables. Since any *combinatory* RCG can be converted into a non-combinatory RCG and we only consider non-combinatory RCGs here, this description suffices.

Generic Architecture

An LPN is a PRCG with the following generic architecture: the *axiom* of an LPN is a unary predicate S, and it has K densely connected binary predicates $\{A_k\}_{k=1}^K$ (Figure 1). For each latent predicate A_k, we define rules such that $A_k(w_a, w_b)$ is true for each possible pair of terminals, $w_a, w_b \in T$. T may include the empty string ϵ, but we do not allow latent predicates of the form $A_k(\epsilon, \epsilon)$: see Section . We also define rules such that $A_k(XY, UV)$ is true for each possible pair of predicates, A_i, A_j, and every possible ordering of the variables X, Y, U and V across A_i and A_j. Finally, we define $S(XY)$ to be the concatenation of the two arguments of $A_1(X, Y)$.

Learning Model

Given a collection of predicates, $\{A_k\}_{k=1}^K$, and a distribution over clauses, $\{\vec{w}_{A_k}\}$, the learning task is to model a set of utterances, $\{x_j\}_{j=1}^J$, as being generated according to the following distribution:

$$\vec{w}_{A_k} \sim Dir(\vec{\alpha}_{A_k})$$
$$x_j \underset{iid}{\sim} p_{\text{PRCG}}(S(x_j) \,|\, \{w_{A_k}\})$$

$$p(\{\vec{w}_{A_k}\} \,|\, \vec{x}, \{\vec{\alpha}_{A_k}\}) \propto$$
$$\prod_j p_{\text{PRCG}}(x_j | \{\vec{w}_{A_k}\}) \prod_{A_k} p_{\text{DIR}}(\vec{w}_{A_k} \,|\, \vec{\alpha}_{A_k})$$

In words, the weights of clauses sharing head predicate A_k are drawn from a Dirichlet distribution defined by $\vec{\alpha}_{A_k}$. Sentences x_j are then drawn from the resulting PRCG.

Bayesian inference over stochastic grammars and stochastic logic programs has been an active area of research in recent decades (Muggleton 2000; Cussens 2001; Liang et al. 2007; Goldwater, Griffiths, and Johnson 2006; Johnson, Griffiths, and Goldwater 2006). Variational inference is a popular approach in this domain and the one we adopt here. As explained in the next section, we implemented inference by translating LPNs into PRISM programs and using its built-in Variational Bayes Expectation-Maximization algorithm (Sato, Kameya, and Kurihara 2008).

Implementation

LPNs can be encoded as a restricted subclass of PRISM programs; this is very similar to how PCFGs are encoded in PRISM (Lloyd et al. 2000). See Figure 2 for an example. A general probabilistic logic programming system based on PROLOG, PRISM provides built-in predicates for probabilistic execution and Bayesian inference over logic programs with stochastic choices. There are several restrictions placed on PRISM programs to maintain the validity of their probabilistic interpretation. Most importantly, derivations of

1) A(a, a) :: 0.25.
2) A(b, b) :: 0.3.
3) A(X a, Y a) ← A(X, Y) :: 0.25.
4) A(X b, Y b) ← A(X, Y) :: 0.2.

(a)

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values('A', [1, 2, 3, 4],
    [0.25, 0.30, 0.25, 0.20]).

reduce('A'-[[a],[a]],1).
reduce('A'-[[b],[b]],2).
reduce('A'-[A2,B2],3) :- lpn('A'-[X,Y]),
    append(X,[a],A2), append(Y,[a],B2).
reduce('A'-[A2,B2],4) :- lpn('A'-[X,Y]),
    append(X,[b],A2), append(Y,[b],B2).

lpn(P-IN) :- reduce(P-IN,V), msw(P,V).
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(b)

Figure 2: Possible encodings of the 2-copy language $\{ww \mid w \in \{a, b\}^+\}$ as (a) an LPN, (b) a PRISM program.

a probabilistic predicate cannot contain cycles. Because we disallow $A_i(\epsilon, \epsilon)$ as a valid clause, every term in the body of a clause has shorter arguments than the head, giving acyclic and finite derivations.

Experiment

To evaluate LPNs as a probabilistic model of concept acquisition, we trained an LPN with 4 latent predicates on a set of sentences expressing successor and predecessor relations in numbers between one and ninety-nine. The training set was the collection of sentences

$$X = \{[\text{after} \mid \text{before}]\ \langle n \rangle\ \text{comes}\ \langle n+1 \rangle \mid n \in 1, \ldots, 99\},$$

where $\langle n \rangle$ is the number word corresponding to n. The lexicon was the set of number word corresponding to 1 through 19, the decades $20, \ldots, 30$, the empty string, and the words "before" and "after." It is not difficult to manually work out an LPN that describes this limited domain of sentences; see Figure 3, for a possible LPN derivation of an example sentence.

Although it is difficult to know how common these kinds of sentences are in child-directed speech, words for small numbers are far more common than words for larger ones (MacWhinney 2000). On the other hand, children learning to count to large numbers rehearse the sequence. To approximate this distribution of evidence, we drew these sample sentences from a sum of a geometric distribution with parameter 0.5 and a uniform distribution. These components were weighted 75% and 25%, respectively. We drew 2000 examples from this distribution, holding out the sentences in Table 2 for evaluation.

For inference, we used default $\frac{1}{D}$ pseudocounts (where D is the dimensionality of the Dirichlet distributions). We found that different random initialization for this experiment did not lead to qualitatively different results, though further investigation will be necessary to see how robust the algorithm is to local maxima when fitting LPNs.

We evaluated the learned model by asking for Viterbi (*i.e.* maximum a posteriori) completions of the last words of each held out test sentence. Table 2 shows these completions. The grammar correctly learns much of the structure of these sentences, including the difference between sentences starting with "before" and "after" and the edge cases that relate decade words like "twenty" to non-decade words like "twenty one."

S(X Y) ← A_1(X, Y) : 1.0000
A_1(X Y, U V) ← A_2(X, U), A_3(V, Y) : 0.5002
A_1(X Y, U V) ← A_3(Y, V), A_4(X, U) : 0.3428
A_1(X Y, U V) ← A_1(V, Y), A_4(X, U) : 0.0796
A_1(X Y, U V) ← A_1(Y, V), A_2(X, U) : 0.0712
A_1(X Y, U V) ← A_2(Y, X), A_3(V, U) : 0.0021
A_1(X Y, U V) ← A_1(V, Y), A_2(X, U) : 0.0013
A_1(X Y, U V) ← A_2(V, X), A_3(Y, U) : 0.0008
A_1(X Y, U V) ← A_1(X, U), A_2(V, Y) : 0.0008
A_1(X Y, U V) ← A_1(X, U), A_2(Y, V) : 0.0008
A_1(X Y, U V) ← A_1(X, U), A_4(Y, V) : 0.0004

A_2(before, comes) : 0.7316	A_4(after, comes) : 0.9990
A_3(one, two) : 0.3993	A_3(two, three) : 0.2063
A_3(three, four) : 0.1093	A_3(four, five) : 0.0734
A_3(five, six) : 0.0502	A_3(six, seven) : 0.0355
A_3(eight, nine) : 0.0290	A_3(seven, eight) : 0.0271
A_2(fifty, fifty) : 0.0375	A_2(thirty, thirty) : 0.0361
A_2(eighty, eighty) : 0.0339	A_3(null, one) : 0.0231
A_2(forty, forty) : 0.0332	A_2(twenty, twenty) : 0.0310
A_2(seventy, seventy) : 0.0296	A_2(sixty, sixty) : 0.0274
A_2(ninety, ninety) : 0.0260	A_3(eighteen, nineteen) : 0.0064
A_3(sixteen, seventeen) : 0.0049	A_3(eleven, twelve) : 0.0044
A_3(nine, ten) : 0.0044	A_3(thirteen, fourteen) : 0.0044
A_3(fourteen, fifteen) : 0.0039	A_3(ten, eleven) : 0.0034
A_2(null, fifty) : 0.0043	A_3(eighty, null) : 0.0030
A_3(seventeen, eighteen) : 0.0030	A_3(nine, sixty) : 0.0025
A_2(nine, seventy) : 0.0029	A_3(nine, forty) : 0.0020
A_2(null, thirty) : 0.0022	A_2(nine, ninety) : 0.0022
A_3(twelve, thirteen) : 0.0015	A_3(fifteen, sixteen) : 0.0015
A_3(null, comes) : 0.0010	A_2(twenty, after) : 0.0014
A_2(sixty, before) : 0.0007	A_3(nineteen, comes) : 0.0005
A_4(nine, thirty) : 0.0010	

Table 1: The 4 predicate LPN trained to model sentences in the number domain. Rules with insignificant weights are removed. This LPN generates the completions in Table 2.

To inspect visually the learned grammar, we thresholded rules according to the expected number of times they were used in parsing the training dataset. Table 1 shows all rules with expected count above $1e-6$. This reduces from 2669 to 52 the number of significant rules. On inspection, predicate A_2 forms "before" sentences, predicate A_4 forms "after" sentences, predicate A_3 is successorship recursively defined over the decades and ones, and predicate A_2 is a category for the decade words.

Our LPN does not learn to how to transition between the last word in a decade and the next decade (e.g. "seventy nine" to "eighty"). Instead, it makes the intuitively reasonable generalization that "seventy nine" should be followed

Question	$K = 4$
after twenty comes ______?	twenty one ✓
after forty five comes ______?	forty six ✓
after forty seven comes ______?	forty eight ✓
after forty nine comes ______?	forty ten ×
after fifty nine comes ______?	fifty ten ×
after sixty one comes ______?	sixty two ✓
after sixty three comes ______?	sixty four ✓
after sixty four comes ______?	sixty five ✓
after sixty five comes ______?	sixty six ✓
after sixty nine comes ______?	sixty ten ×
after seventy three comes ______?	seventy four ✓
after seventy nine comes ______?	seventy ten ×
after ninety five comes ______?	ninety six ✓
before twenty three comes ______?	twenty two ✓
before thirty comes ______?	thirty eighty ×
before thirty eight comes ______?	thirty seven ✓
before forty one comes ______?	forty ✓
before fifty three comes ______?	fifty two ✓
before sixty eight comes ______?	sixty seven ✓
before seventy two comes ______?	seventy one ✓
before seventy three comes ______?	seventy two ✓
before eighty five comes ______?	eighty four ✓
before ninety two comes ______?	ninety one ✓
before ninety three comes ______?	ninety two ✓
before ninety five comes ______?	ninety four ✓

Table 2: Viterbi completions of held-out sentences to evaluate an LPN with four latent predicates in the number domain. The LPN achieves an accuracy of $\%80$.

$$S(\text{after twenty five comes twenty six})$$
$$S(XY) \leftarrow A_1(X, Y).$$
$$\uparrow$$
$$A_1(\text{after twenty five, comes twenty six})$$
$$A_1(XY, UV) \leftarrow A_2(X, U),\ A_3(Y, V).$$
$$A_2(\text{after, comes}). \qquad A_3(\text{twenty five, twenty six})$$
$$A_3(XY, UV) \leftarrow A_4(X, U),\ A_5(Y, V).$$
$$A_4(\text{twenty, twenty}). \qquad A_5(\text{five, six}).$$

Figure 3: A possible parse of the sentence "after twenty five comes twenty six" using a 5-predicate LPN.

by "seventy ten."

Discussion

Though simple, the knowledge learned by LPNs in the number domain we explore in this paper goes beyond the expressive capacity of PCFGs and HMMs, and yet it avoids intractability by using a restricted formalism. The human ability to learn abstract concepts must also rely on such a compromise, and it remains to be seen to what extent LPNs can model this ability.

This work leaves many open questions. What domains of concepts are amenable to being represented and learned by LPNs? Can multiple and varied systems of concepts be learned at once? Perhaps most importantly, are there generic architectures for LPNs that allow a larger number of predicates without an insurmountable blow-up in the number of rules? One idea which we are pursuing is using layered connectivity, much like neural networks, to limit the number of rules in which each predicate participates.

Many of the algorithms and formalisms used here were originally developed for use in other closely related areas, including logic programming and semantic parsing. Semantic parsing, in particular, seems especially related to the challenge we face here of jointly learning meaning and structure from sentences (Berant et al. 2013; Liang, Jordan, and Klein 2013; Kwiatkowski et al. 2010; Poon and Domingos 2009). Semantic parsing, however, tends to frame this challenge slightly differently than we do here, namely, by asking how utterances can be mapped to an explicit internal logical language. By contrast, we focus here on systems where meaning and structure seem inseparable. Understanding how these two approaches relate and inform one another is an interesting and open question.

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